Spring 2014 **Math 601** Name: ____________________

Quiz 4

Question 1. (8 pts)

Determine whether the following statements are true or false. If false, explain why.

(a) V is a vector space of dimension n and S is a set of vectors in V. If S spans V, then the number of vectors in S is $\geq n$.

Solution: True.

(b) Let W be the subspace spanned by $\{1, t\}$ in $P_9(t)$. Then $\dim W = 2$.

Solution: True.

(c) Let u and $v_1, v_2, \cdots, v_m$ be vectors in a vector space V. If u is a linear combination of $v_1, \cdots, v_m$, then the set $\{u, v_1, \cdots, v_m\}$ is linearly dependent.

Solution: True.

(d) Let V be a vector space of dimension n. If $\{u_1, \cdots, u_m\}$ is linearly independent set of vectors in V, then $m = n$.

Solution: False.

Question 2. (12 pts)

(a) Given vectors

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 4 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ -3 \\ -6 \\ 3 \end{bmatrix}, v_4 = \begin{bmatrix} 4 \\ -1 \\ 8 \\ 1 \end{bmatrix}$$

Determine whether v_1, v_2, v_3 and v_4 are linearly independent.

Solution: For the matrix

$$A = \begin{bmatrix} | & | & | & | \\ v_1 & v_2 & v_3 & v_4 \\ | & | & | & | \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 3 & 4 & -6 & 8 \\ 0 & -1 & 3 & 1 \end{bmatrix}$$

Use Gaussian elimination to find its reduced echelon form

$$\begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

whose rank is 2. So v_1, v_2, v_3 and v_4 are linearly dependent.

(b) Let

$$A = \begin{bmatrix} | & | & | & | \\ v_1 & v_2 & v_3 & v_4 \\ | & | & | & | \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 3 & 4 & -6 & 8 \\ 0 & -1 & 3 & 1 \end{bmatrix}$$

Find a basis for the column space of A.

Solution: Notice that the matrix A is the same as that in Part (a). Using the reduced echelon form from Part (a), we see that a basis of the column space of A is

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 4 \\ -1 \end{bmatrix}$$